Vertex on, inside, and outside the circle

In this lesson we'll look at three combinations of angles and circles and how the angle measures are related to the measures of the intercepted arcs.

Here we look at three possible combinations for the location of the vertex of an angle that intercepts a circle.

Inside of the circle

When the vertex of the angle is on the inside of the circle, two vertical angle pairs are formed. Remember that vertical angles are congruent. The relationship of the intercepted arcs and the angle is that the sum of the measures of the intercepted arcs, divided by 2, is equal to the measure of one of the angles. Notice this is also called the angle formed by two intersecting chords.

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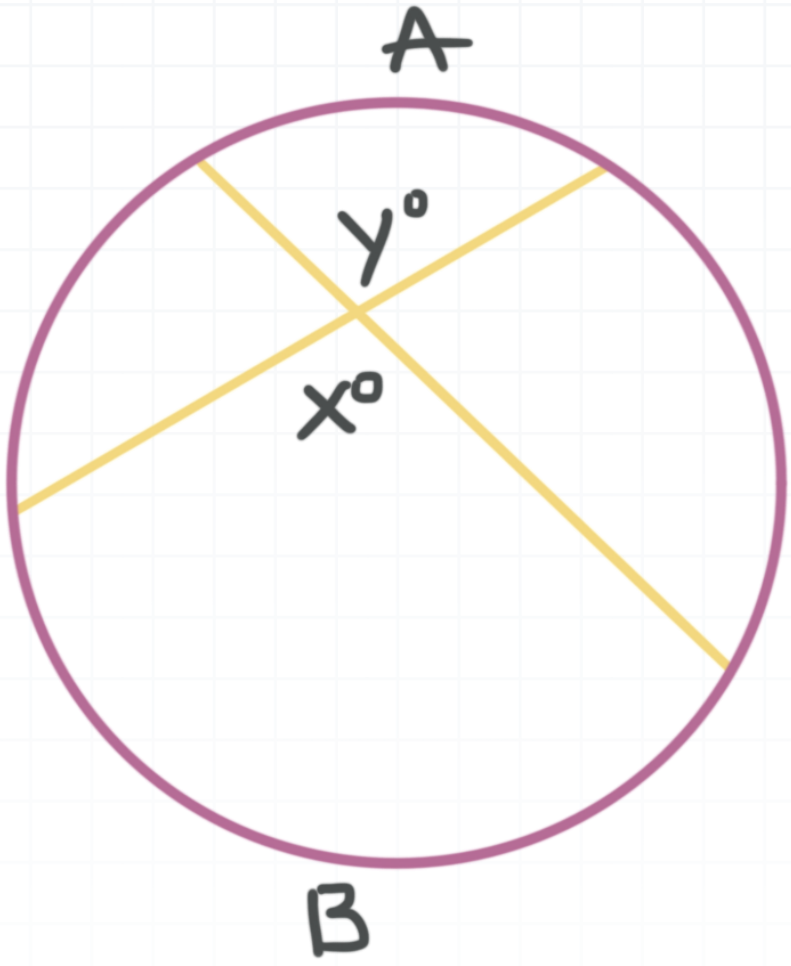

$$x^\circ = y^\circ$$

$$x^\circ = \frac{m\overset{\frown}{A} + m\overset{\frown}{B}}{2}$$

$$y^\circ = \frac{m\overset{\frown}{A} + m\overset{\frown}{B}}{2}$$

On the circle

When the vertex of the angle is on the circle, then the angle is called an **inscribed angle**. Remember from the last lesson that an inscribed angle is equal to the measure of its intercepted arc, divided by 2.

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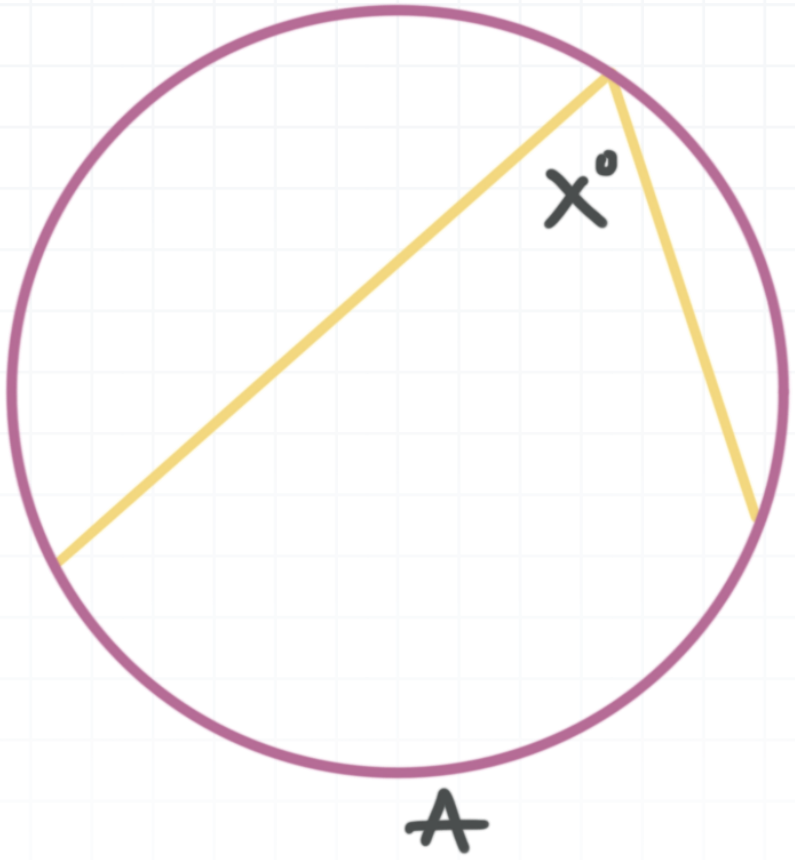

$$x^\circ = \frac{m\overset{\frown}{A}}{2}$$

Outside the circle

When the vertex of the angle is outside of the circle, the measure of the angle is equal to $1/2$ of the difference of the intercepted arcs. There are three possibilities for this to occur, but the formula is the same for all of them.

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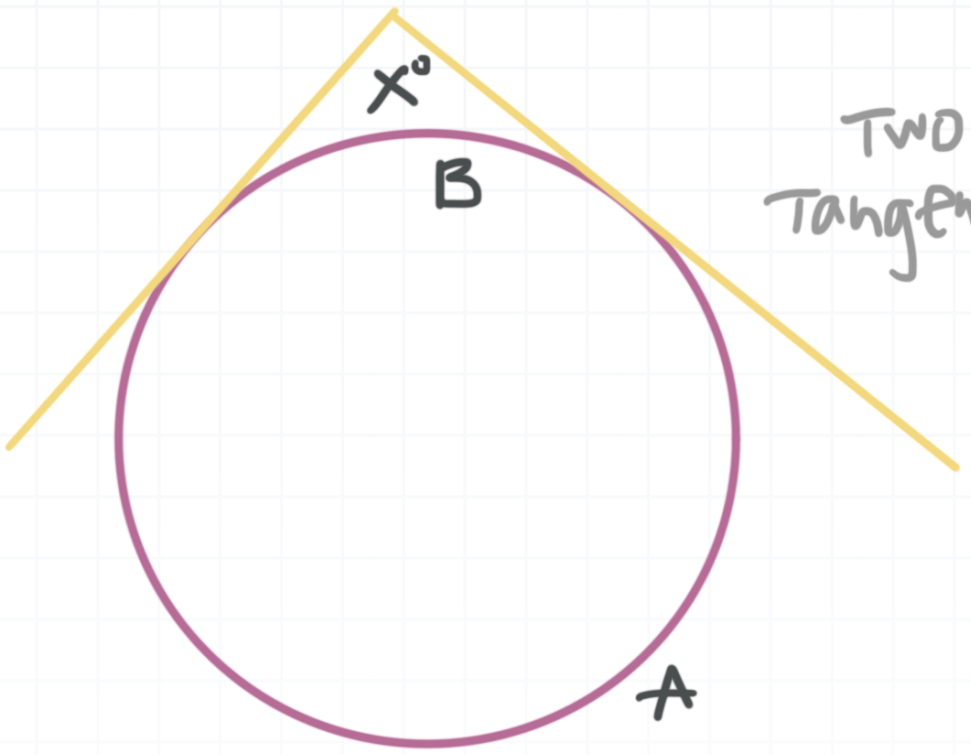
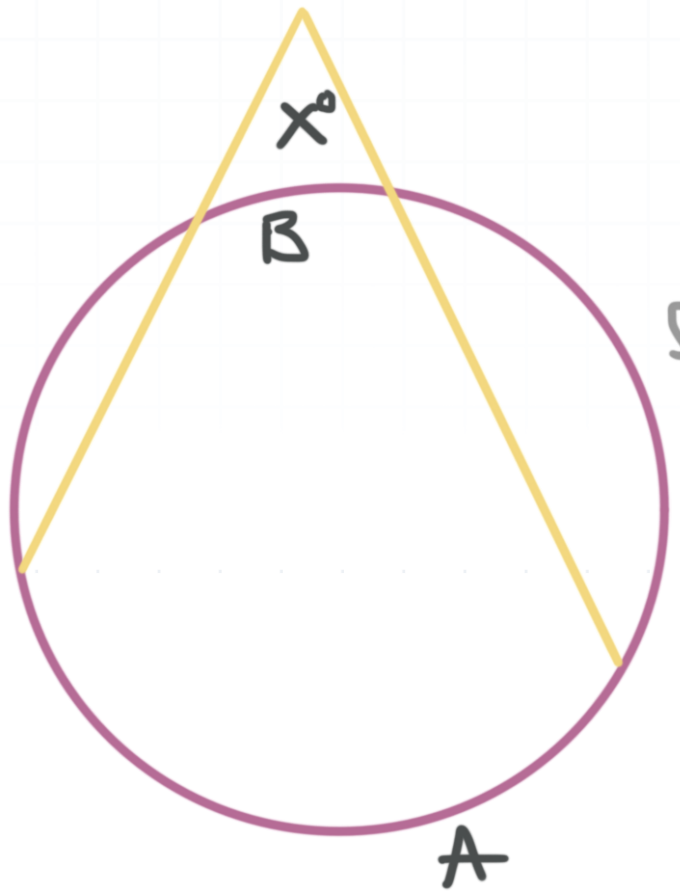


Two Tangents



Two secants

$$x^\circ = \frac{m\overset{\frown}{A} - m\overset{\frown}{B}}{2}$$

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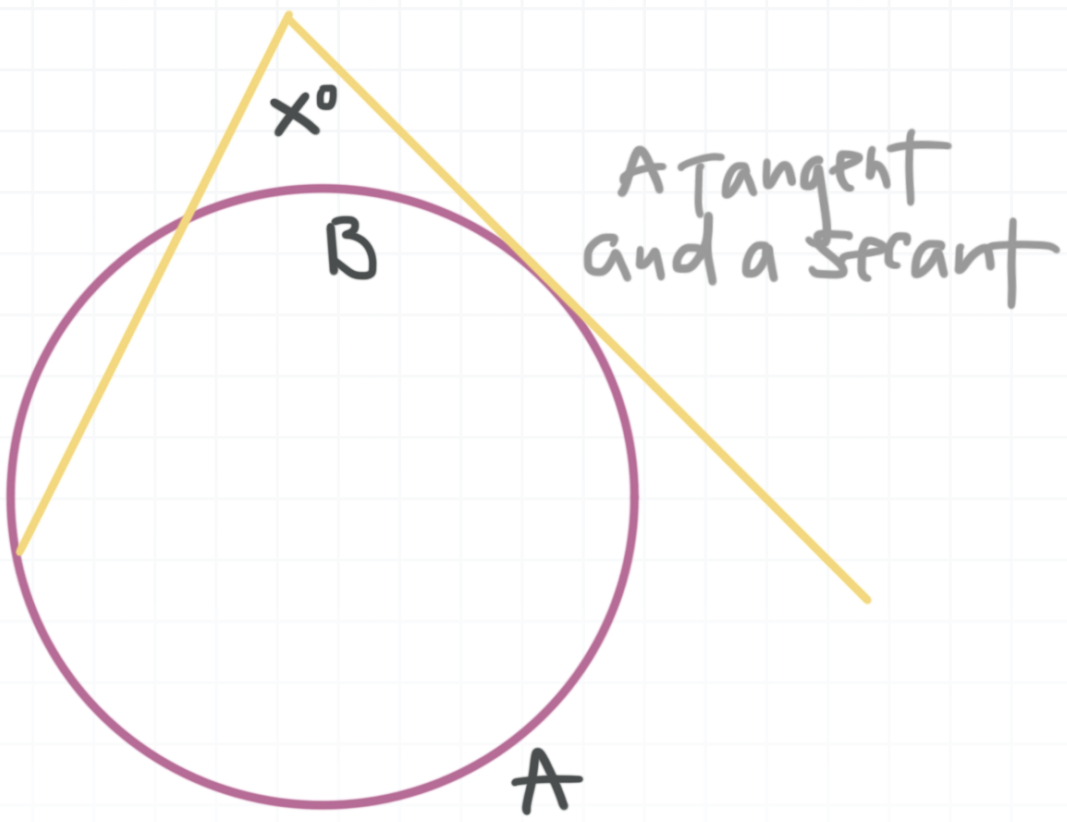
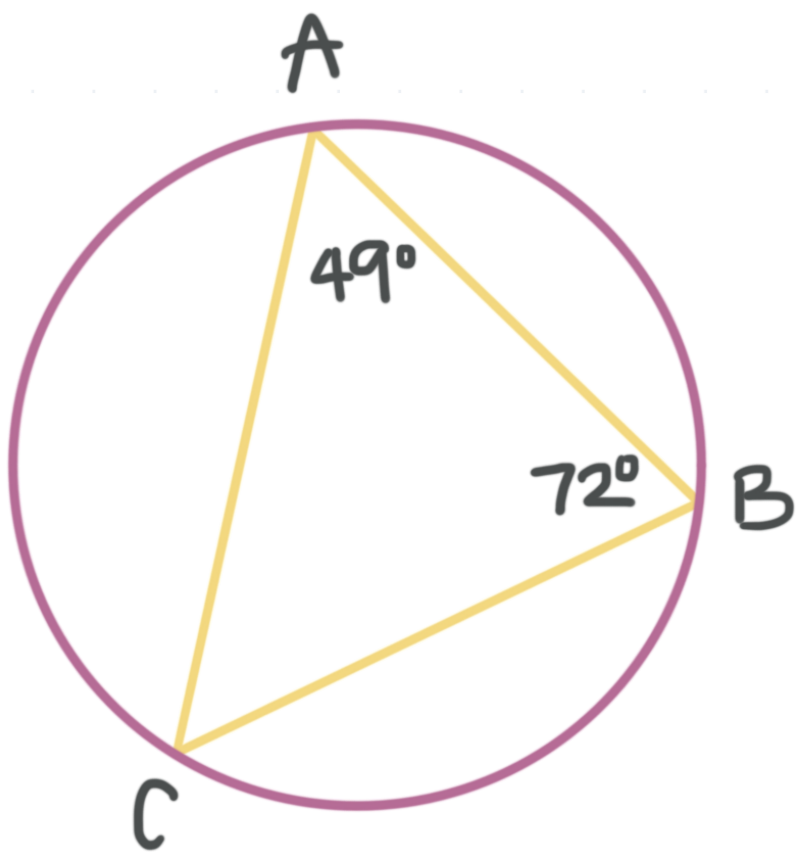


Let's start by working through an example.

Example

What is the measure of arc AB?



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Arc AB is the intercepted arc of angle ACB. $\angle ACB$ is an inscribed angle, so it's half of the measure of $\overset{\frown}{AB}$. We know that the measures of the angles in a triangle add to $180°$, so we can find $m\angle ACB$ and use it to find the measure of $\overset{\frown}{AB}$.

$$m\angle ACB + 72° + 49° = 180°$$

$$m\angle ACB + 121° = 180°$$

$$m\angle ACB = 59°$$

And we know that

$$m\angle ACB = \frac{1}{2} m\overset{\frown}{AB}$$

So

$$2m\angle ACB = m\overset{\frown}{AB}$$

$$2(59°) = m\overset{\frown}{AB}$$

$$118° = m\overset{\frown}{AB}$$

Let's do two more examples.

Example

What is the value of x?

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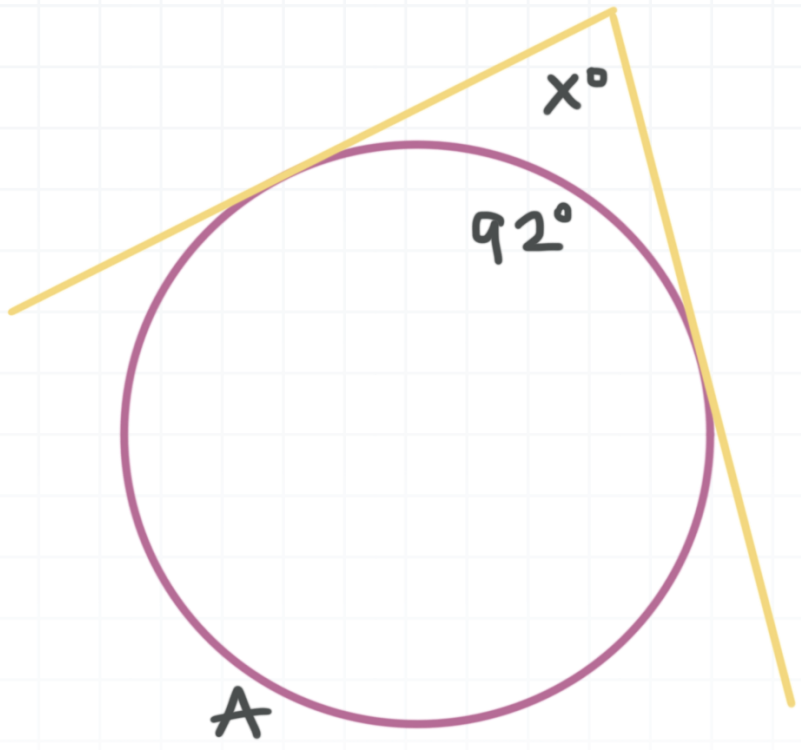

x is the measure of an angle formed by two tangent lines to a circle. The measure of the angle is the difference of the intercepted arcs, divided by 2.

We know the measure of one arc is $92°$, so we need to find the measure of arc A to use the formula. A full circle has $360°$, so we can find the measure of arc A by subtracting $92°$ from $360°$.

$$m\widehat{A} = 360° - 92°$$

$$m\widehat{A} = 268°$$

Now we can use the formula.

$$x° = \frac{268° - 92°}{2}$$

$$x° = \frac{176°}{2}$$

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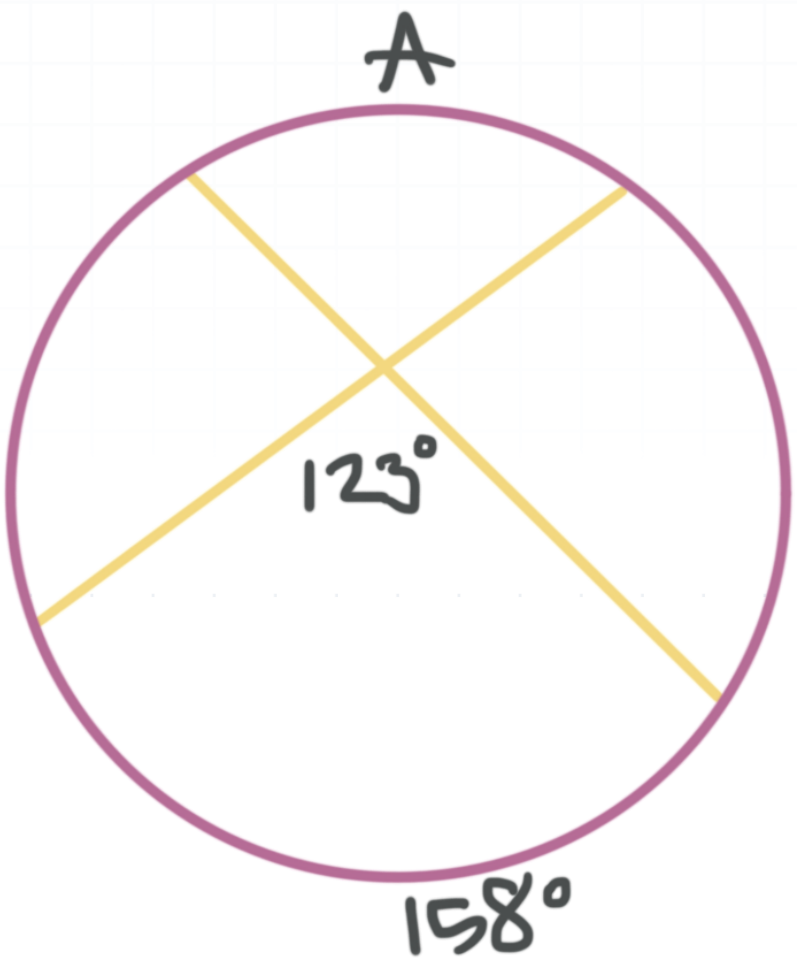
$$x^\circ = 88^\circ$$

$$x = 88$$

Let's try one with a vertex inside the circle.

Example

What is the measure of arc A?



We know that the sum of the intercepted arcs, divided by 2, is equal to the angle. So we can set up the equation and then solve for $m\widehat{A}$.

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$$123^\circ = \frac{158^\circ + m\overset{\frown}{A}}{2}$$

$$2(123^\circ) = 158^\circ + m\overset{\frown}{A}$$

$$246^\circ = 158^\circ + m\overset{\frown}{A}$$

$$m\overset{\frown}{A} = 88^\circ$$

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